Parallel and Distributed Algorithms and Programs
TD n°6 - Nested Loops (2)

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All documents are available on my website: `http://hadriencroubois.com/#Teaching`

Part 1

Lamport's hyperplane method

Let's consider the following code:

for $i = 1$ *to* N **do**
  for $j = 1$ *to* N **do**
    S_1: $a(i, j) \leftarrow b(i, j - 6) + d(i - 1, j + 3)$
    S_2: $b(i + 1, j - 1) \leftarrow c(i + 2, j + 5) + 1$
    S_3: $c(i + 3, j - 1) \leftarrow a(i, j + 2)$
    S_4: $d(i, j - 1) \leftarrow a(i, j - 1) - 1$

Question 1

a) List all dependencies and build the dependency matrix.

b) Describe the domain as a polytope, ie give A and B such that:

$$Dom = \{p : Ap \leq B\}$$

c) Explain how the dependency matrix characterizes an order on the vertices in that polytope.

d) What condition must a linear schedule π satisfy in order to respect the dependencies ? (Bonus: give a proof)

e) Find such a schedule and use it to parallelize the code.

Part 2

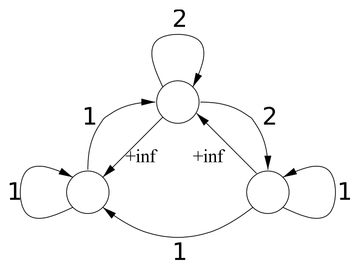
Allen and Kennedy algorithm

Question 2

a) Considering the following code, give the reduced dependency graph, and for each dependency, give its type (flow, anti, output) and its level. Apply the Allen and Kennedy algorithm to restore the nested loops and verify the nature of the obtained loops.

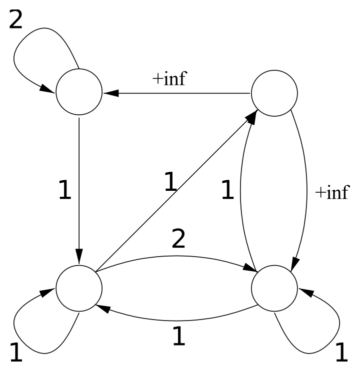
for $i = 1$ *à* N **do**
  for $j = 1$ *à* N **do**
    S_1: $a(i + 1, j + 1) \leftarrow a(i + 1, j) + b(i, j + 2)$
    S_2: $b(i + 1, j) \leftarrow a(i + 1, j - 1) + b(i, j - 1)$
    S_3: $a(i, j + 2) \leftarrow b(i + 1, j + 1) - 1$

b) Give perfect nested loops, where all the dependencies are uniform, and of which the RDG is exactly the following:



parallelize it using the Allen and Kennedy algorithm.

c) Give perfect nested loops, where all the dependencies are uniform, and which RDG is exactly the following:



parallelize it using the Allen and Kennedy algorithm.